**Using the Solow model to study Climate Change**

**Deadlines:**

**-Round 1 (spreadsheet simulations): 15 April, 08:30**

**-Round 2 (pdf report): 6 May 08:30**

**Both rounds must be submitted via Canvas**

Use the student ANRs for naming the files. E.g., EGI\_2022\_A1\_0123\_32355\_2322.pdf (.xlsx) is the first assignment, submitted by the students with ANRs 0123, 32355 and 2322.

Table. Student information

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
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*Instructions*: The assignment consists of two rounds. In the first round you are asked to numerically simulate the model. In the second round you are asked to submit a report with the interpretation of the numerical results. This second-round report consists of the questions marked with **[R2]** in this file, they should be answered here [as indicated below] and should be submitted as a .pdf.

The points for each question are provided in the rubric of Round 2 in Canvas.

The spreadsheet is not graded, but **the Round 1 submission is a pre-requisite for the Round 2 report. If you do not submit the spreadsheet with your calculations by the Round 1 deadline, your Round 2 Report will not be graded. Your Round 1 submission should demonstrate that you attempted to perform ALL the required simulations. Incomplete Round 1 submissions will receive no feedback.**

We model the inter-relation between economic activity and climate change at a global scale. A by-product of economic activity is the emission of Green House Gases (GHG). Emissions increase the concentration of GHG in the atmosphere, which results in higher temperatures. In turn, the climate change unraveled by global warming has detrimental effects for economic activity; we refer to the latter as damages.

The model consists of two modules. An economic module which follows from the Solow model with technological progress and models the dynamic evolution of output and GHG emissions. This is complemented with a climate module that models the relationship between GHG concentrations and temperature. From the economic module we obtain GHG emissions that feed the GHG concentrations in the climate module. From the climate module we obtain temperature which in turn determines the damages suffered by the economy in the form of lost output.

Economic module

Material output (1)

Productivity (2)

Population (3)

GDP: Output net of damages (4)

Consumption (5)

Physical capital accumulation (6)

GHG emissions (7)

Climate module

Accumulation of GHG in the atmosphere (8)

Temperature increase (relative to 1800) (9)

Damages as a proportion of (10)

Where:

investment rate in physical capital

investment rate in emission reduction

population growth rate

rate of technological progress

emission intensity without investment in emission reduction

efficiency of investment in emission reduction

sensibility of emissions to investment in emission reduction

sensibility of temperature to GHG concentrations

and sensibility of damages to temperature

You have access to an excel file with multiple spreadsheets.

**All the relevant parameter values and initial values are provided in the excel file**

It is your task to perform different simulations of the Solow model with climate detailed above (Round 1). Based on these simulations you will analyze the economic implications of climate change and climate policy scenarios (Round 2).

Each simulation is to be performed on a separate sheet of the excel file (see the sheet labels)

Present all your answers using 4 decimals.

*\*Note: For the computations of growth rates use the exponential formulation. For ‘a’ variable this is:*

*where is the growth rate of variable between year and year .*

1. **[R2]** Use equation (7) to explain why is a measure of the effort to reduce emissions (i.e., abatement effort).

Equation 7 states: . To test the hypothesis that is a measure of the effort to reduce emissions one must take the first order constraint of with respect to . This can be done as follows:

Since , , , , (which means that ) and thus is also larger than 0, the F.O.C is less than 0 (due to the negative sign at the front). Hence, as we increase we would observe a decrease in the value of .

1. **[R2]** Is there free lunch when it comes to abatement effort ? If not, to what is this economy ‘sacrificing’ in order to increase ?

There is no free lunch, increasing

* Use the information above and the parameters in the excel file to simulate the population, , and productivity, , up to 2100 in the corresponding columns of the three simulation sheets (sim1, sim2, sim3). That is, the series of population and productivity are the same across simulations. Take as given the initial levels (i.e., the 2020 values) of and as provided in the excel file.

**Simulation 1 - Baseline scenario [*no abatement effort*]**

*[Perform this simulation using the sheet labeled sim1]*

You will start with the simulation of a baseline scenario where there is no abatement effort.

Computing initial values

* Throughout this simulation we assume that the abatement effort is equal to zero for all the years between 2020 and 2100. Complete the series of accordingly (*column G*)

In what follows the goal is to obtain the series of all the remaining endogenous variables () up to 2100.

Here you are presented with a 5-steps procedure to produce the dynamic simulation of the model. These steps are meant as a guide.

* + Step 1: Use equation (1) to compute the 2020 value of . Use this and the initial level of abatement to compute the level of emissions in 2020 according to equation (7).
  + Step 2: Use the 2020 value of GHG concentrations, , to compute the 2020 value of the temperature increase with respect to 1800, , according to equation (9). Use this and equation (10) to compute the 2020 damages, .
  + Step 3: Compute the 2020 levels of GDP per capita, , (*column M*) and consumption per capita, , (*column O*).
  + Step 4: Use the dynamic equation (6) to compute the immediately subsequent value of physical capital. Use the dynamic equation (8) to compute the immediately subsequent value of the atmospheric concentrations of GHG ().
  + Step 5: Repeat Steps 2 to 3 to calculate the immediately subsequent value for each of the other variables (). Repeat Step 4 to calculate the immediately subsequent value of physical capital and of GHG concentrations (, ). Use this procedure to complete the series up to 2100 for the endogenous variables and .
* Compute the yearly growth of GDP per capita, , (*column N*) and consumption per capita, , (*column P*).

1. **[R2]** Suppose for a moment (only for this question) that climate plays no role in the model; that is, think of the Solow model with technological progress covered in the first half of the course. Given the parameter values proposed, what would be the growth rate of GDP per capita in steady state? Explain

Since the long run steady state value of under these conditions (this was found by solving for the steady state in efficiency units and multiplying this by ), the growth rate in steady state can be found, which is:

.

Hence, the long run growth rate of GDP per capita is g, which has a value of 0.02.

1. **[R2]** Present a line chart of the growth rate of GDP per capita, computed in *column N*, as a function of time for the 2021-2100 period.

Figure 1:

Chart, line chart

Description automatically generated

1. **[R2]** Describe and explain what you observe in the figure. To what do you attribute this time-path of the growth rate of GDP per capita between 2021 and 2100?

The growth rate of GDP per capita has a downward sloping tend. In fact, with the addition of climate change, the growth rate goes below the steady state growth rate if we exclude climate change. This could be because the value of Y and hence y depends on the state of the economy. If climate change is worse and more capital gets destroyed, then the value of Y and thus y is bound to decrease.

In this simulation the abatement effort () takes a value of 0, indicating that the government is not allocating any effort towards climate control. Hence, it is only natural that the emissions increase, and the climate situation worsens, decreasing the growth rate of y and creating the trend illustrated in Figure 1.

1. **[R2]** Compare the last decade of the yearly growth rate of GDP per capita computed in *column N* with the long-run growth rate that you would have obtained in the model without climate. Do these rates differ? If so, to what do you attribute the difference?

The last value (2100) has a growth rate of 0.00476, or if rounded: 0.005. This is ¼th of the value found in question b. This difference can once again be attributed to the abatement effort of the government. Since this value is 0, the climate keeps worsening (as more emissions are produced) and the value of damages (D(t)) which depends positively on T(t) which also depends positively on M(T) (emissions) increases. An increase in the value of D(t) decreases the economy’s ability to grow and creates this growth rate that is below the long run growth rate of an economy without a climate parameter.

**Simulation 2 – Maximum abatement**

*[Perform this simulation using the sheet labeled sim2]*

* According to equation (7), if is equal to emissions are fully abated. Throughout this simulation we assume that the abatement effort is equal to for all the years between 2020 and 2100. Complete the series of accordingly (*column G*).
  + Apply the same procedure used in Simulation 1 (Steps 1-5) to complete the series for all the remaining endogenous variables () up to 2100.
* Compute the yearly growth of GDP per capita, , (*column N*) and consumption per capita, , (*column P*).

1. **[R2]** Compare the growth rate of GDP per capita and consumption per capita. Describe and explain the observed relationship between these two growth rates.

[Answer here]

1. **[R2]** Start from the capital accumulation equation and the assumption that is constant at a level that fully abates emissions. Use your knowledge of the Solow model to derive analytical expressions (function only of parameters, initial levels of damages and technology, and time) for the growth rate and the level of GDP per capita in steady state,   and . Include a sufficiently detailed step-by-step of your derivations.
2. **[R2]** Present a line chart of the growth rate of GDP per capita, computed in *column N*, as a function of time for the 2021-2100 period.

[Insert figure here]

1. **[R2]** Describe and explain what you observe in this figure; link your explanation to your answers to literals c and h.

[Answer here]

**Simulation 3 – Incremental abatement**

*[Perform this simulation using the sheet labeled sim3]*

According to equation (7), if is equal to emissions are fully abated. In this simulation we assume that the economy starts from a low level of abatement effort. This effort is gradually increasing over time. For any year between 2020 and 2100 the abatement effort is governed by the following equation

(11)

with and (values provided in the excel file)

1. **[R2]** Use equation (11) to show that
   1. The abatement effort is increasing over time
   2. In the long run the economy approaches to full abatement

[Answer here]

* Complete the series of according to equation (11) (*column G*).
  + Apply the same procedure used in Simulation 1 (Steps 1-5) to complete the series for all the remaining endogenous variables () up to 2100.

**Evaluation of results**

*[Perform the following exercises in the sheet labeled ‘evaluation’]*

You will start with the evaluation of the simulations 2 and 3 relative to the baseline (simulation 1) based on the yearly outcomes for consumption per capita.

* [Optional]: Link *columns C-E* in the *evaluation* sheet to the series of consumption per capita that you obtained in the three simulations: i.e., Link *col C* to *col M in sim1*, *col D* to *col M in sim2*, *col E* to *col M in sim 3*.
* For each year between 2020 and 2100 compute difference between consumption per capita under simulation 2 and under simulation 1. We refer to this as the *Current Value* (CV) difference.
* For each year between 2020 and 2100 compute the *Present Value* (PV), in terms of year 2020, of the difference between consumption per capita under simulation 2 and under simulation 1. *Use the continuous compounding formulation of the PV, where the PV of a variable in year is*: ; where is the discount rate; its value is provided in the excel file.
* For each year between 2020 and 2100 compute the *Current Value* (CV) of the difference between consumption per capita under simulation 3 and under simulation 1.
* For each year between 2020 and 2100 compute the *Present Value* (PV) of the difference between consumption per capita under simulation 3 and under simulation 1. Use the same formula and discount rate as in the comparison between simulations 2 and 1.

1. **[R2]** Based on the results of the CV of the difference in consumption per capita, is consumption per capita under full abatement (simulation 2) always higher than under no abatement (simulation 1)? If not, when is consumption per capita higher under full abatement? Explain your results based on the short-run/long-run costs and benefits of a higher abatement effort.

[Answer here]

1. **[R2]** Would your results change if you consider the PV instead? Explain.

[Answer here]

Now you will evaluate the simulations based on the aggregation of the yearly outcomes. For this consider the following measure, that we will refer to as the *Cumulative Present Value* ():

(12)

The formula above implies that for given the CPV sums the PVs of all the preceding years up to and including year itself: i.e., the sum of the PVs from the initial year up to year .

1. **[R2]** Interpret the following statement: If a decision maker (e.g., the government, voter,…) uses the CPV as described in (12) to evaluate evolution of a variable , then defines how many years into the future this decision maker cares about.

[Answer here]

In what follows we refer to in (12) as the “planning horizon”

* For each year between 2020 and 2100 compute the CPV up to that year. For example, the value of the CVP in 2050 requires using in (12).

1. **[R2]** Use equation (12) and your results for the PV of the difference in consumption per capita between simulation 2 and 1, and between simulation 3 and 1, to complete the following table [*use three decimals*]

**Table 1: CPV**

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| 20 | 2040 |  |  |
| 40 | 2060 |  |  |
| 60 | 2080 |  |  |
| 80 | 2100 |  |  |

Where the denotes consumption per capita under simulation .

1. **[R2]** Describe and explain the evolution of each of the two CPVs in Table 1 (sim2-sim1 and sim3-sim1) as the planning horizon, , is extended. According to these results, would myopic decision makers (i.e., decision makers with a relatively short planning horizon ) support high abatement efforts ? Explain.

[Answer here]

1. **[R2]** Describe and explain the differences between each of the two CPV columns in Table 1; consider the differences in the time path of abatement effort between simulations 2 and 3 in your answer.

[Answer here]

**Taking stock (critical assessment of the model)**

1. **[R2]** What are the main merits of model that we developed in this assignment to study the economic implications of climate change and climate policy?

[Answer here]

1. **[R2]** What are the main limitations of model? Propose a direction in which you would extend/alter the model to deal with these limitations.

[Answer here]